# ROSEVILLE COLLEGE

## YEAR 12

## EXTENSION 1 MATHEMATICS

## JUNE 2005 ASSESSMENT

**Time allowed: 45 minutes+ 2 minutes reading time**

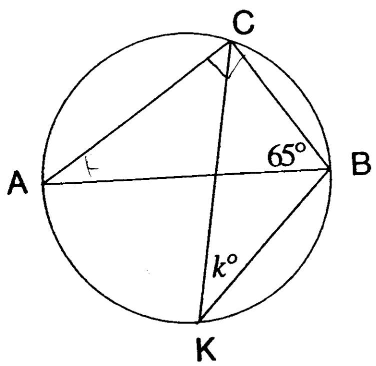
**DIRECTIONS TO CANDIDATES:**

- **Attempt all questions**
- **All questions are of equal value. The part marks for each section are shown on the right hand side of the page**
- **Please start each question on a new page.**
- **Staple the questions separately**
- **All necessary working should be shown. You may not be awarded the marks for an answer unsupported by working.**

**Question 1** (12 marks) **Marks**

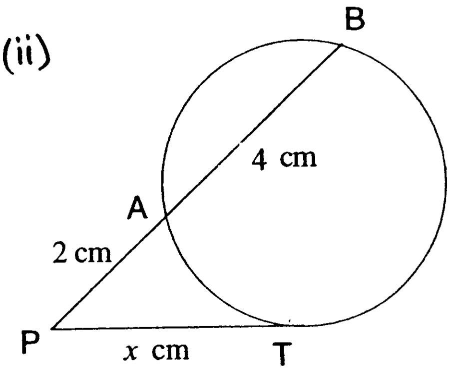
a) Find the value of the pronumeral in each of the following. You do not have to give reasons.

(i)



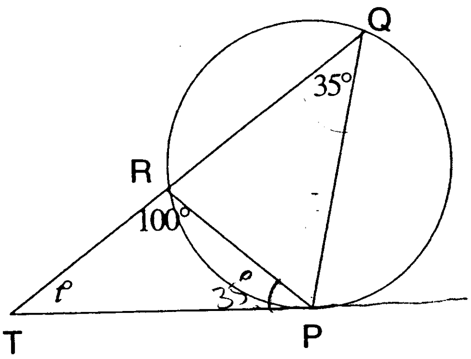

AB is a diameter: $\angle ABC = 65°$,
$\angle BKC = k°$
Find the value of $k$

(ii)




PT is the tangent at T; PA = 2 cm,
PT = $x$ cm and AB = 4 cm. Calculate
in simplest form, the exact value of $x$

(iii)




TP is the tangent at P;
$\angle PRT = 100°$; $\angle PQT = 35°$
Find the value of $t$. (3)

2u-> b) In a fish hatchery, the fish population, N, satisfies the equation $N = N_0 e^{kt}$ where $N_0$ and $k$ are constants and $t$ is measured in months.

(i) Initially there were 1 000 fish in the hatchery and at the end of 5 months there were 5000. Find the value of $k$ correct to 3 decimal places. (2)

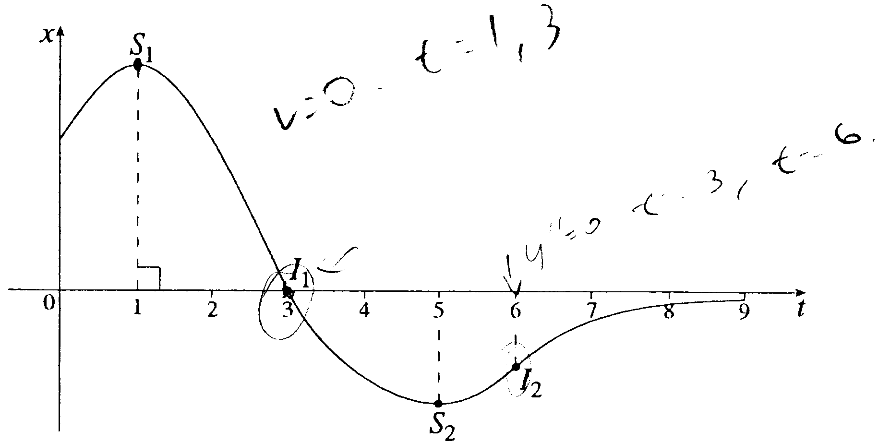
(ii) Find the number of fish in the hatchery at the end of 8 months. (Give your answer correct to the nearest hundred.) (1)

(iii) At the end of which month will the fish population exceed 50 000 for the first time? (1)

(iv) At what rate is the population increasing at the end of six months? (Give your answer correct to the nearest hundred fish per month.) (2)

6month.
$t=6$.

2M
c)



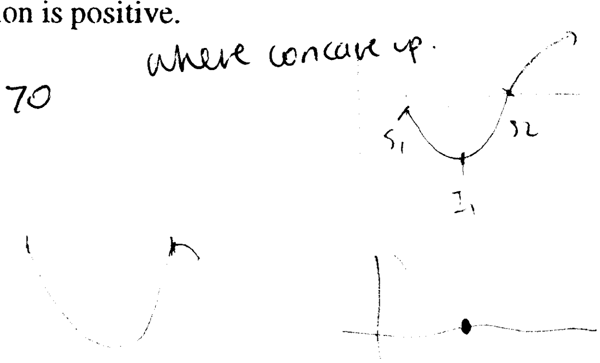

The graph shows the position of a particle, moving on a straight line, over the first nine seconds of the motion.

$S_1$ and $S_2$ are stationary points; $I_1$ and $I_2$ are points of inflexion.

(i) State the times, or periods of time, for which a>0.

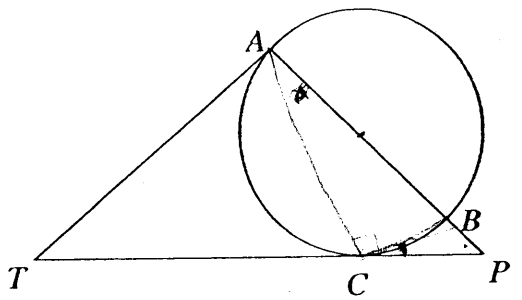
1. the particle is stationary (1)
2. the velocity is negative (1)
3. the acceleration is positive. (1)

$y'' > 0$ where concave up.



**Question2 (12 marks)** **Marks**

**a)**




(FIGURE NOT TO SCALE)

*AB* is a diameter of a circle *ABC*. The tangents at *A* and *C* meet at *T*. The lines *TC* and *AB* are produced to meet at *P*. Copy the diagram into your examination booklet. Join *AC* and *CB*.

i) Prove that $< CAT = 90 - < BCP$. (2)

ii) Hence, or otherwise, prove that $< ATC = 2 < BCP$. (2)

**b)** A particle moves along a straight line about a fixed point $O$ so that its velocity, $v$ ms$^{-1}$, at time $t$ seconds is given by $v = 4\sin(2t + \frac{\pi}{6})$.

Initially the particle is $\sqrt{3}$ metres to the left of $O$.

i. Find expressions for the displacement, $x$, of the particle at any time $t$. (2)

ii. At what time does the particle first return to its initial position? (2)

**c)** A spherical balloon is being inflated at the rate of 1000 cm$^3$s$^{-1}$. You are given that $V = \frac{4}{3}\pi r^3$ and $A = 4\pi r^2$.

(i) Show that $\frac{250}{\pi r^2}$ is an expression for the instantaneous rate of change of the radius. (2)

(ii) Find the rate of change of the surface area of the balloon when the radius is 10 cm. (2)

**Question 3** (12 marks) **Marks**

a) A particle moving in a horizontal straight line is performing Simple Harmonic Motion. At time $t$ seconds its displacement $x$ metres from a fixed point $O$ on the line is given by $x = 3\cos 2t + \sin 2t$, where displacements to the right of $O$ are positive.

Explain whether the particle is initially moving to the right or to the left, and whether it is speeding up or slowing down. (2)

b) A particle is moving in simple harmonic motion with a velocity (in m/s) given by $v^2 = 2 - x - x^2$ where $x$ is the displacement ( in metres) from a point O.

(i) What are the end points of the particles oscillation? (1)

(ii) Find the maximum velocity of the particle. (1)

c) A particle moves in a straight line so that at time $t, (t \geq 0)$ its acceleration $a$, is given by

$$a = 4x$$

where $x$ is the displacement of the particle from the origin. The particle starts its journey one metre to the right of the origin $(\text{at } x = 1)$ with a velocity of $v = -2$.

$t=0, x=1.$
$v=-2$

(i) Show that $v = -2x$. (2)

(ii) Express $x$ as a function of $t$. (2)

(iii) Explain whether or not the particle ever moves to the left of the origin. (1)

$v=0 \quad x=6$

d) A point moving with simple harmonic motions starts from rest at a point 6 cm from the centre of the motion. If the point has a speed of 10 cm/s when it passes through the centre of motion, find the period of the motion. (3)

6 6

QUESTION 3

a) $x = 3\cos 2t + \sin 2t$

$v = -6\sin 2t + 2\cos 2t$.

$t=0$, $v = -6\sin 0 + 2\cos 0$

$= 2$ $\frac{1}{2}$

$\therefore$ moving to the right. $\frac{1}{2}$

$\ddot{x} = -12\cos 2t - 4\sin 2t$

$t=0$, $\ddot{x} = -12\cos 0 - 4\sin 0$

$= -12$ $\frac{1}{2}$

$v$ and $\ddot{x}$ have opposite signs

$\therefore$ slowing down. $\frac{1}{2}$

b) i) $v^2 = 2 - x - x^2$

$0 = (2+x)(1-x)$. ($v=0$ at end points.)

$x = -2, 1$

ends points are $-2$ and $1$.

ii) max $v$ when at centre. ie. $x = -\frac{1}{2}$

$v^2 = 2 - (-\frac{1}{2}) - (-\frac{1}{2})^2$

$= 2 + \frac{1}{2} - \frac{1}{4}$

$= 2\frac{1}{4} = \frac{9}{4}$

$v = 3/2$ m/s.

c) i) $\ddot{x} = 4x$

$\frac{d}{dx}(\frac{1}{2}v^2) = 4x$ $\frac{1}{2}$

$\frac{1}{2}v^2 = 2x^2 + c$ $\frac{1}{2}$

$v = -2$, $x = 1$, $\frac{1}{2}(-2)^2 = 2(1)^2 + c$

$2 = 2 + c$

$c = 0$ $\frac{1}{2}$

$\frac{1}{2}v^2 = 2x^2$

$v^2 = 4x^2$

$v = \pm\sqrt{4x^2}$

$= \pm 2x$ but $v < 0$ when $x = 1$ $\frac{1}{2}$

$\therefore v = -2x$

ii) $v = -2x$

$\frac{dx}{dt} = -2x$

$\frac{dt}{dx} = \frac{1}{-2x}$ $\frac{1}{2}$

$t = -\frac{1}{2}\ln x + c.$ $\frac{1}{2}$

$t=0,\ x=1,\quad 0 = -\frac{1}{2}\ln 1 + c.$

$c = 0.$ $\frac{1}{2}$

$t = -\frac{1}{2}\ln x$

$-2t = \ln x$

$e^{-2t} = x.$ $\frac{1}{2}$

iii) If $t>0,\ e^{-2t}>0$

$\therefore$ No.

d) $x = a\cos(nt+\alpha)$ $\qquad t=0,\ v=0,\ x=6$

$t=0,\ x=6,\quad 6 = a\cos\alpha.$

$v = -an\sin(nt+\alpha)$

$t=0,\ v=0\quad 0 = -an\sin\alpha.$

$0 = \sin\alpha.$

$\alpha = 0$

$\therefore\ 6 = a\cos 0$

$6 = a \times 1$

$a = 6$

$\therefore\ x = 6\cos nt$

$v = -6n\sin nt$

$x=0,\ v=10,\quad 0 = 6\cos nt$

$0 = \cos nt \ \therefore\ nt = \frac{\pi}{2}, \frac{3\pi}{2}\ \therefore\ t = \frac{\pi}{2n}, \ldots$

$10 = -6n\sin nt$

$10 = -6n\sin n.\frac{\pi}{2n}$

$10 = -6n\sin\frac{\pi}{2}$

$10 = -6n \times 1$

$\frac{10}{-6} = n$

$n = \frac{5}{3}\ \therefore$ Period $= \frac{2\pi}{5/3} = \frac{6\pi}{5}$